EE 230
Lecture 14

Basic Feedback Configurations

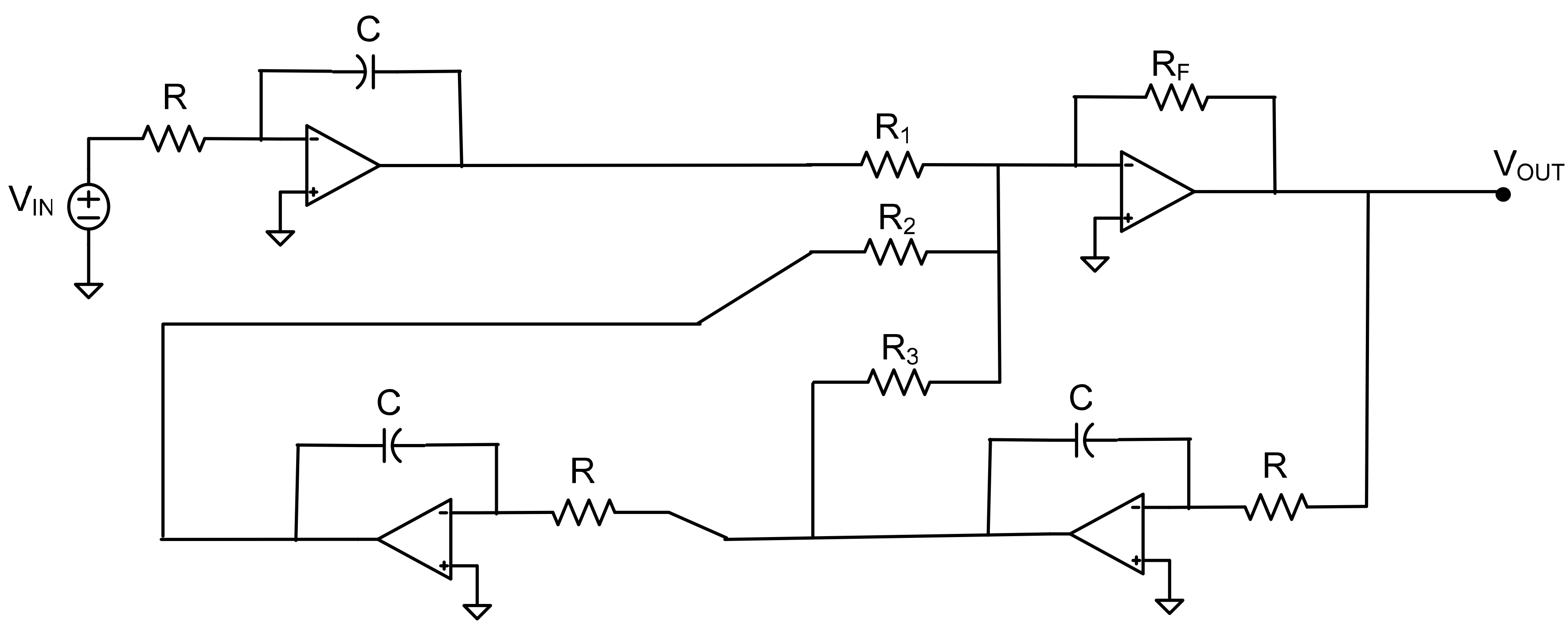
- Second-Order Filters
- Difference Amplifiers
- Impedance Converters

Quiz 10

a) Determine the transfer function $T(s)=V_{OUT}(s)/V_{IN}(s)$ for the circuit shown

b) Is the circuit stable?

Assume the op amps are ideal and all resistors are 1Ω and all capacitors are 1F



And the number is ?

1 3 8

5 4

?

2 6

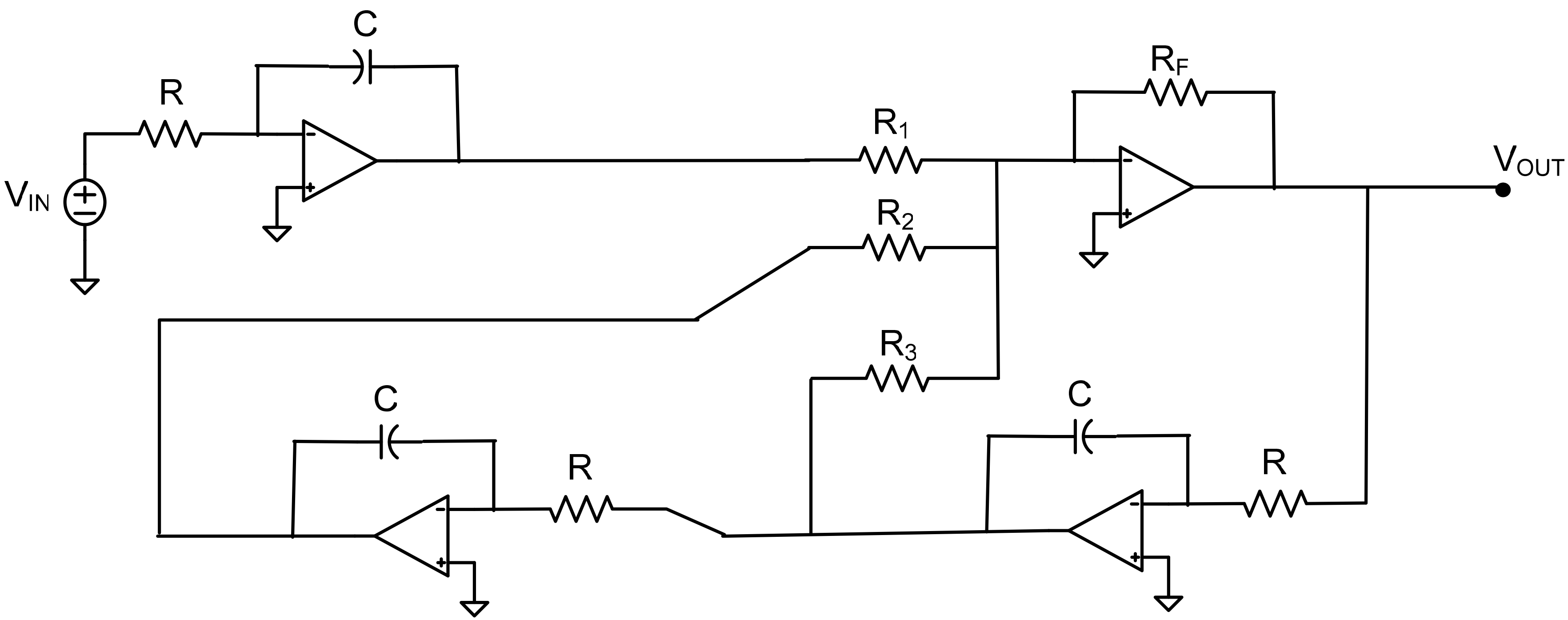
9 7

Quiz 10

Solution:

a) Determine the transfer function $T(s)=V_{OUT}(s)/V_{IN}(s)$ for the circuit shown

b) Is the circuit stable?

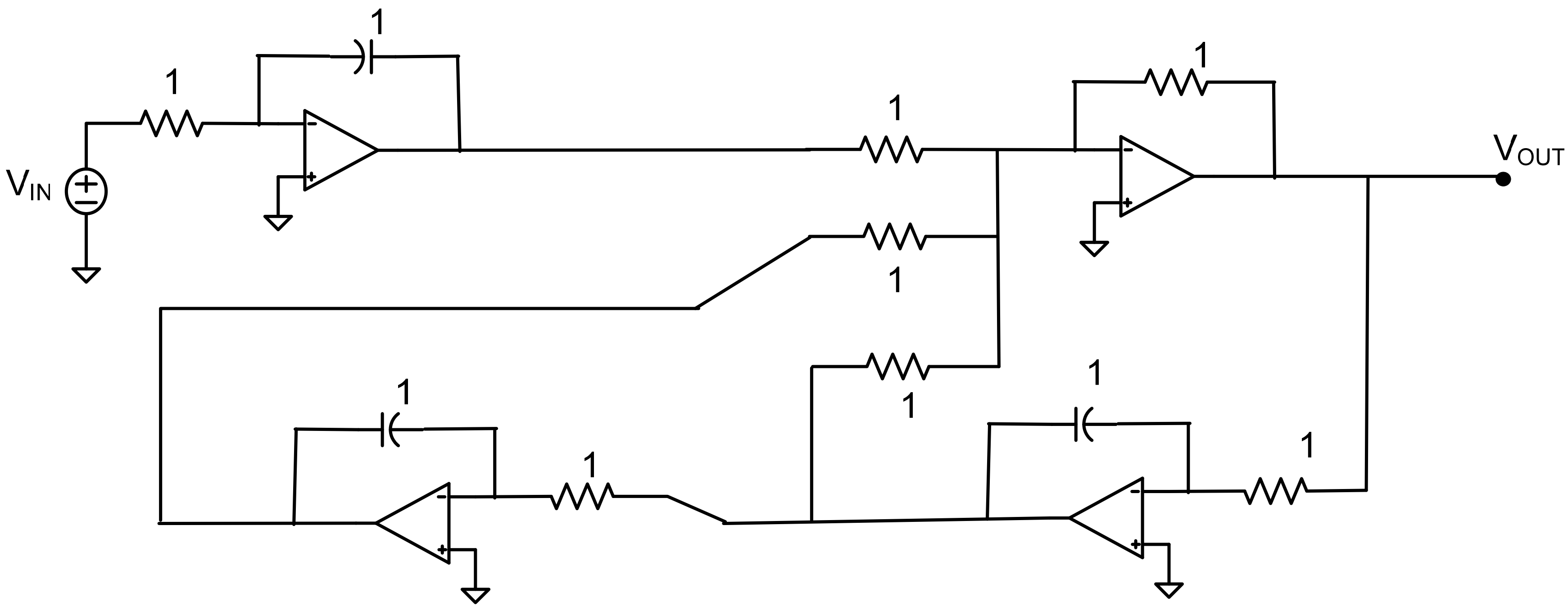


Quiz 10

Solution:

a) Determine the transfer function $T(s)=V_{OUT}(s)/V_{IN}(s)$ for the circuit shown

b) Is the circuit stable?



Quiz 10

Solution:

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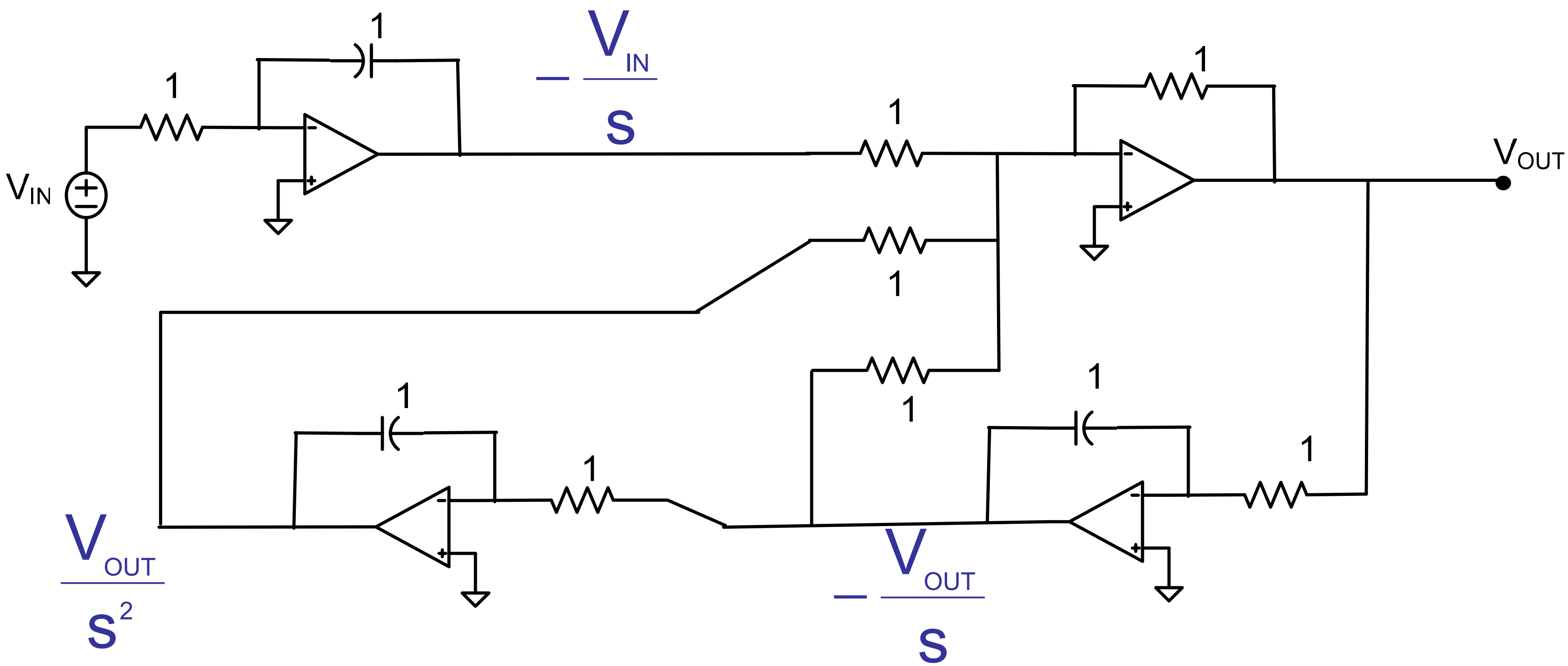


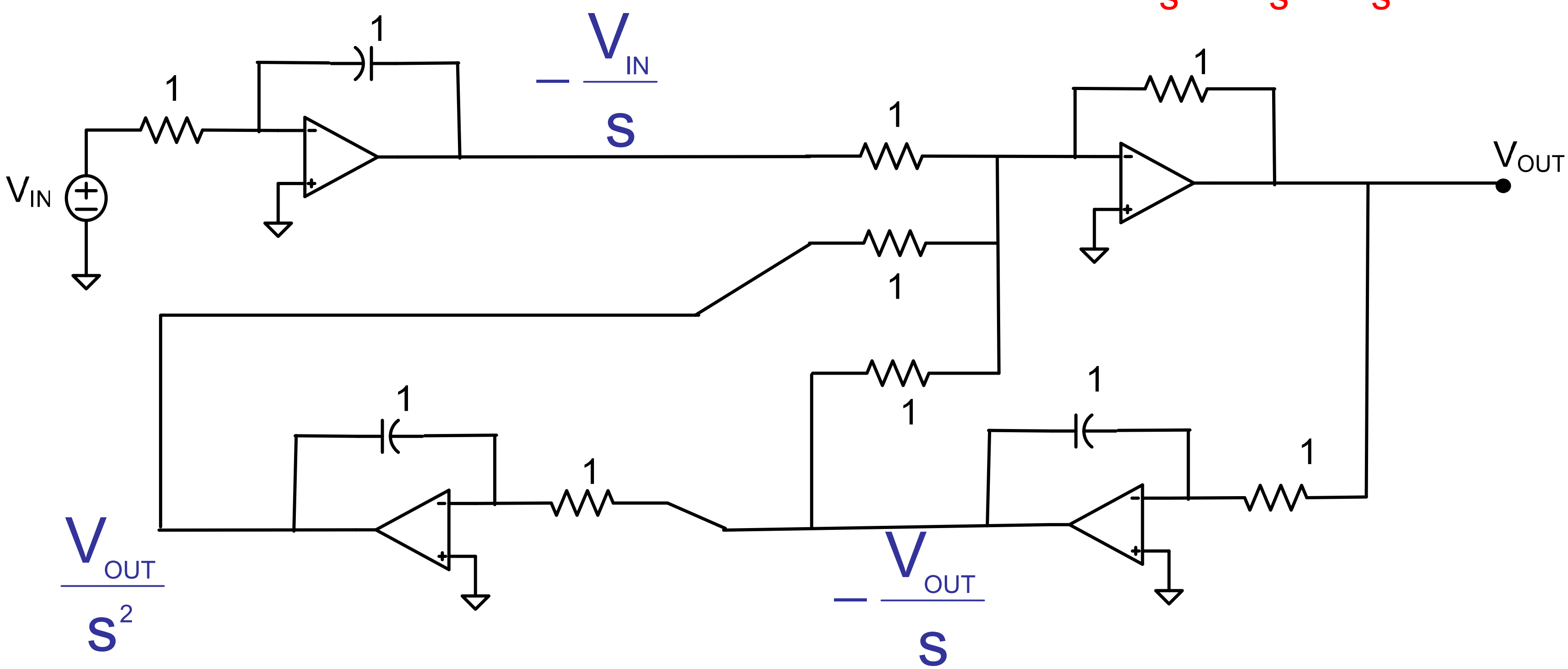
Quiz 10

Solution:

a) Determine the transfer function $T(s)=V_{OUT}(s)/V_{IN}(s)$ for the circuit shown

b) Is the circuit stable?

$$V_{OUT} = -\frac{V_{IN}}{s} - \frac{V_{OUT}}{s} - \frac{V_{OUT}}{s^2}$$



Quiz 10

Solution:

a) Determine the transfer function $T(s)=V_{OUT}(s)/V_{IN}(s)$ for the circuit shown

b) Is the circuit stable?

$$V_{OUT} = -- \frac{V_{IN}}{s} -- \frac{V_{OUT}}{s} - \frac{V_{OUT}}{s^2}$$

$$s^2V_{OUT} - sV_{OUT} + V_{OUT} = sV_{IN}$$

$$V_{OUT}\left(s^2 - s + 1\right) = sV_{IN}$$

$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{s}{s^2 - s + 1}$$

Quiz 10

Solution:

a) Determine the transfer function $T(s)=V_{OUT}(s)/V_{IN}(s)$ for the circuit shown

b) Is the circuit stable?

$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{s}{s^2 - s+1}$$

Poles at

$$s = \frac{1+j\sqrt{3}}{2} \qquad s = \frac{1-j\sqrt{3}}{2}$$

Quiz 10

Solution:

a) Determine the transfer function $T(s)=V_{OUT}(s)/V_{IN}(s)$ for the circuit shown

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$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{s}{s^2 - s + 1}$$

Poles at

$$s = \frac{1+j\sqrt{3}}{2} \qquad s = \frac{1-j\sqrt{3}}{2}$$

RHP

RHP

Quiz 10 Solution:

a) Determine the transfer function $T(s)=V_{OUT}(s)/V_{IN}(s)$ for the circuit shown

b) Is the circuit stable?

$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{s}{s^2-s+1}$$

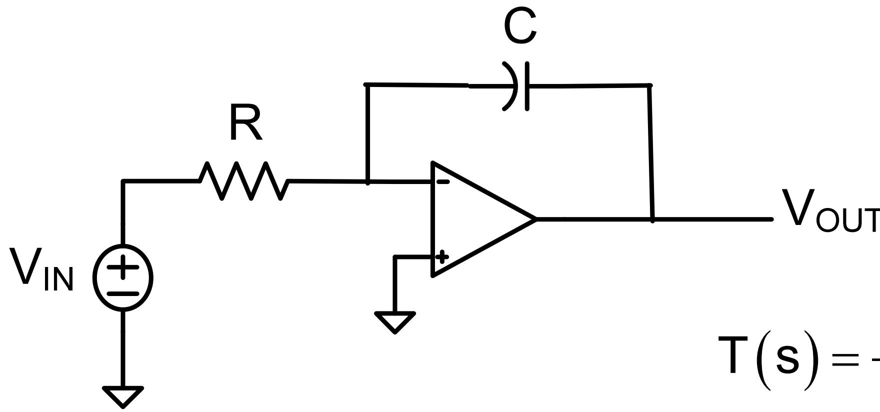
Poles at

$$s = \frac{1+j\sqrt{3}}{2} \qquad s = \frac{1-j\sqrt{3}}{2}$$

RHP LHP

Since there is one RHP pole, the circuit is unstable !

Inverting Integrator



$$T(s) = -\frac{1}{sRC}$$

$$T(j\omega) = -\frac{1}{j\omega RC}$$

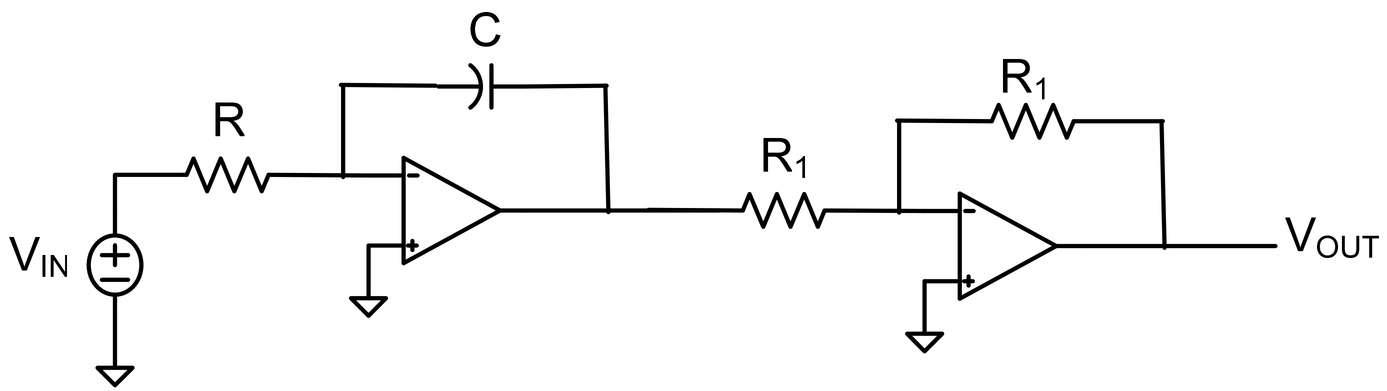
$$\left|T(j\omega)\right| = \frac{1}{\omega RC}$$

$$\angle T(j\omega) = 90^o$$

Unity gain frequency is $\omega_0 = \dfrac{1}{RC}$

Review from Last Time

Noninverting Integrator



$$V_{OUT} = \frac{1}{RC}\int_0^t V_{IN}(\tau)\,d\tau + V_{IN}(0)$$

Obtained from inverting integrator by preceding or following with inverter

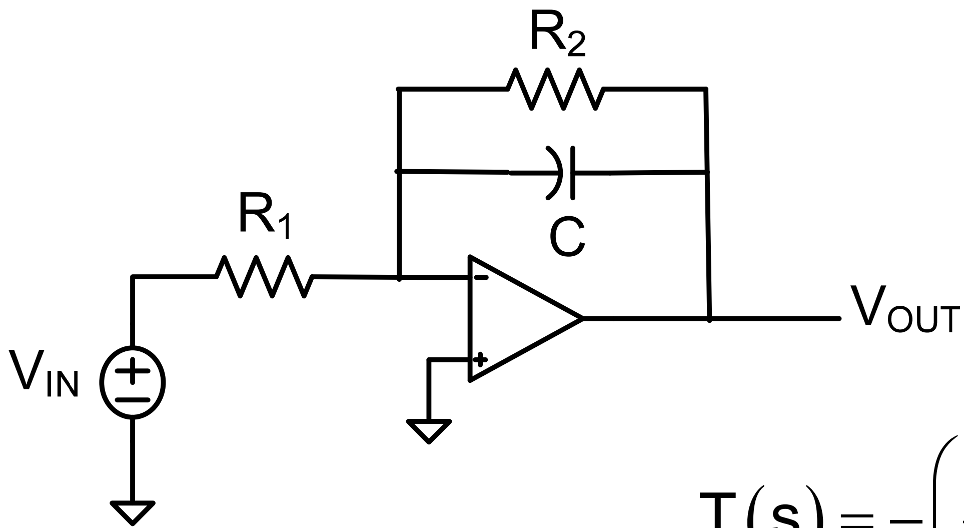
Requires more components

Also widely used

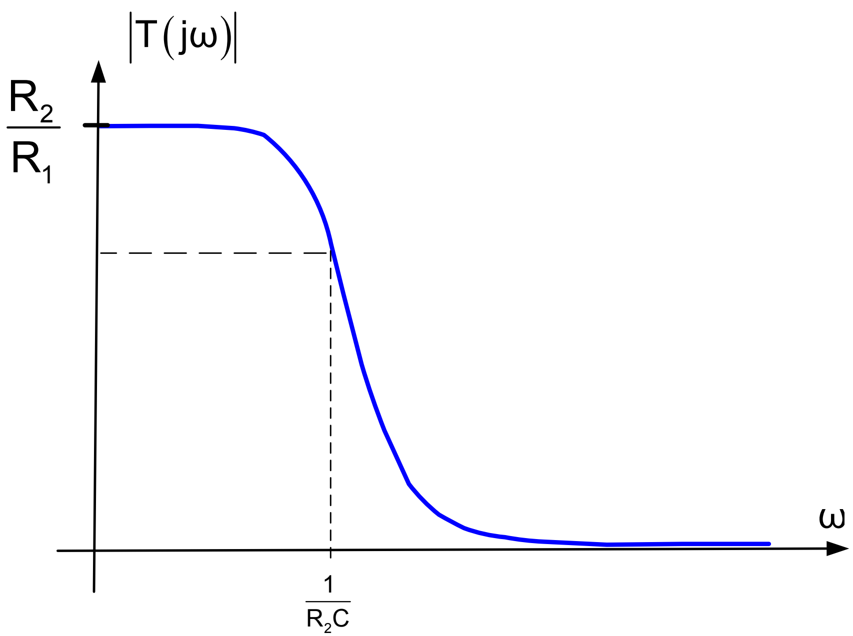
Same issues affect noninverting integrator

Review from Last Time

First-order lowpass filter with a dc gain of R_2/R_1



$$T(s) = -\left(\frac{R_2}{R_1}\right)\frac{1}{1+sCR_2}$$

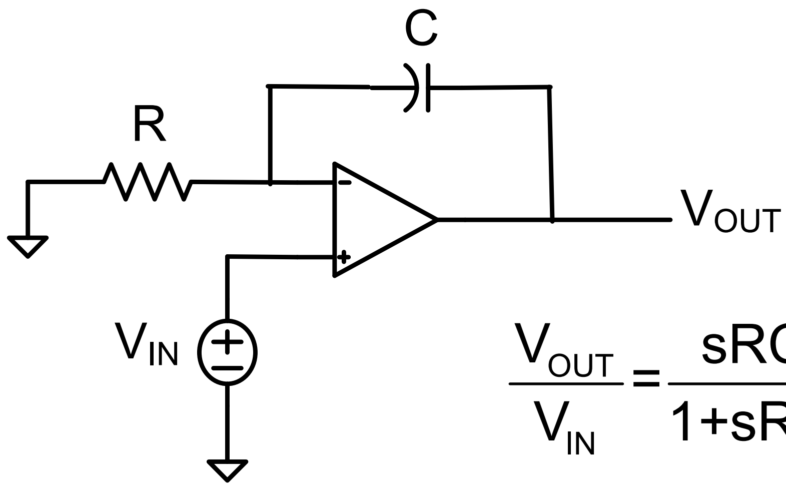


R_2 controls the pole
(and also the dc gain)

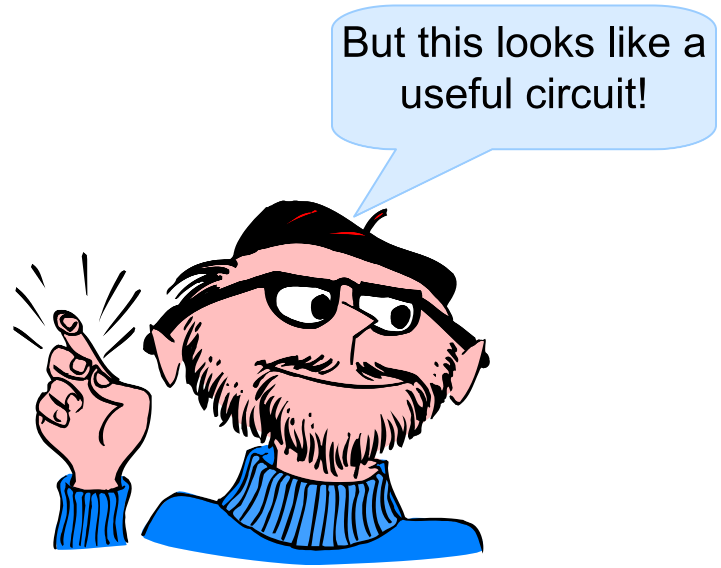
R_1 controls the dc gain
(and not the pole)

Review from Last Time

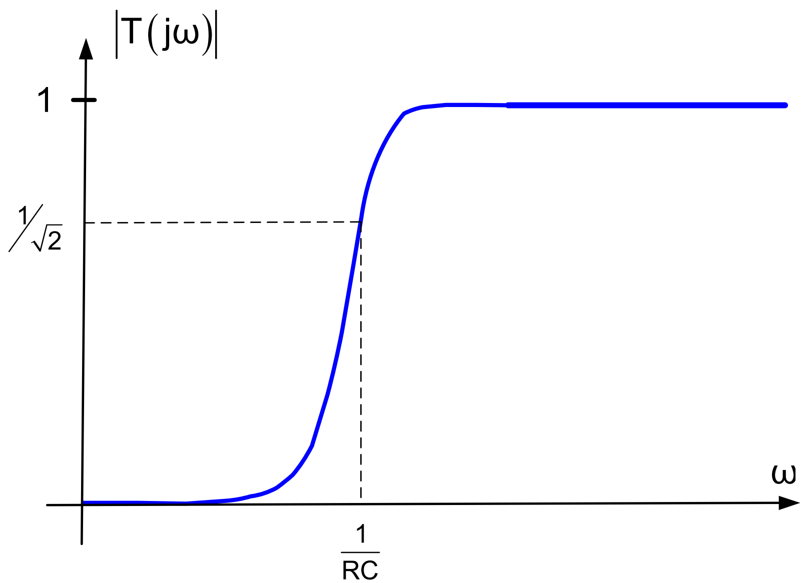
First-Order Highpass Filter



$$\frac{V_{OUT}}{V_{IN}} = \frac{sRC}{1+sRC}$$



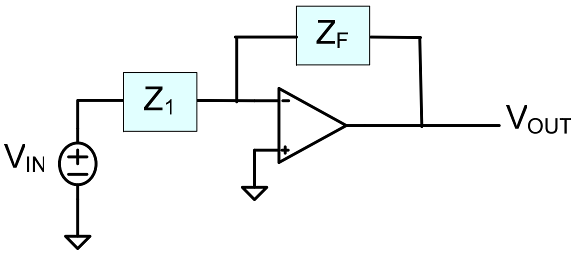
This is a first-order high-pass amplifier (or filter)

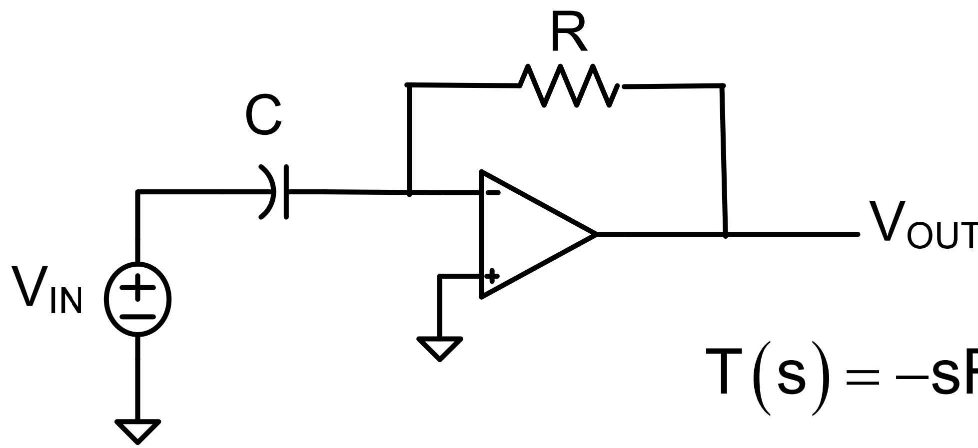


3dB band edge at ω=1/(RC)

Review from Last Time

Inverting Differentiator





$$T(s) = -sRC$$

Differentiator gain ideally goes to ∞ at high frequencies

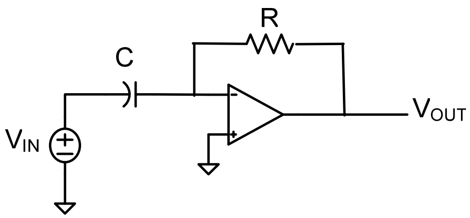
Differentiator not widely used

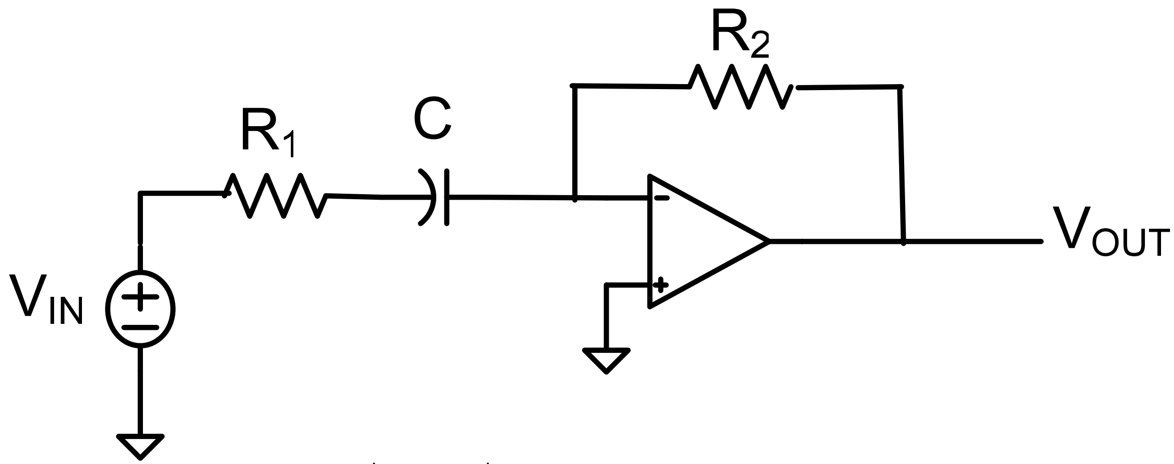
Differentiator relentlessly amplifies noise

Stability problems with implementation (not discussed here)

Placing a resistor in series with C will result in a lossy differentiator that has some applications

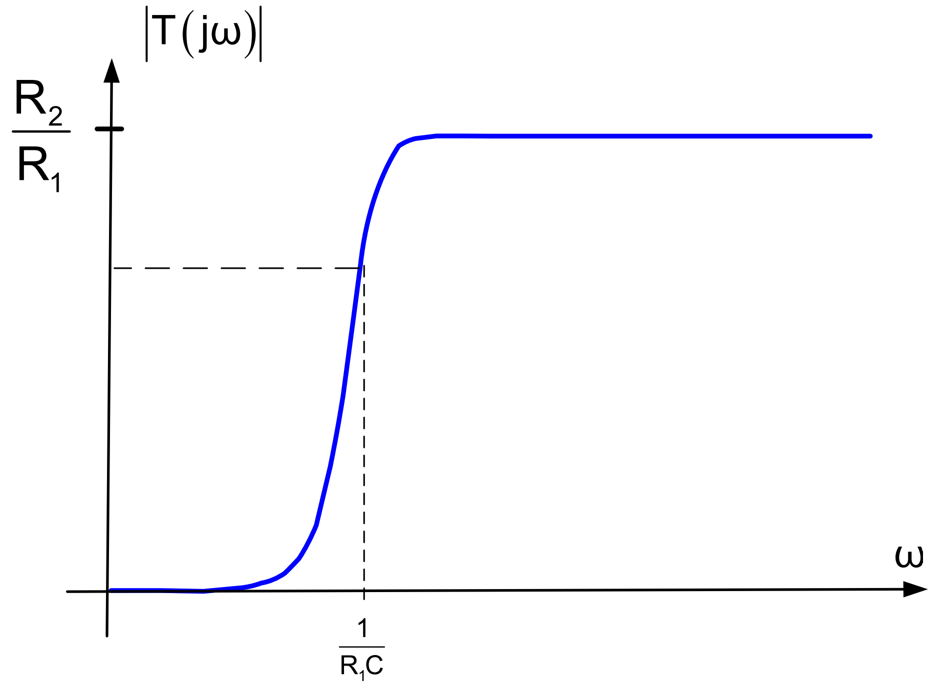
First-order High-pass Filter





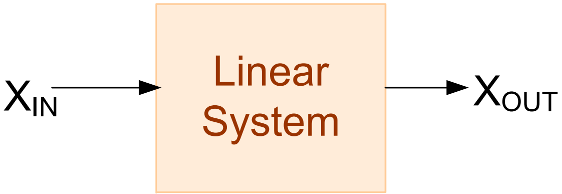
$$T(s) = -\frac{R_2}{R_1 + \frac{1}{sC}} = -\frac{sR_2C}{1+R_1Cs}$$

$$\left|T(j\omega)\right| = \frac{\omega R_2 C}{\sqrt{1+\left(\omega R_1 C\right)^2}}$$



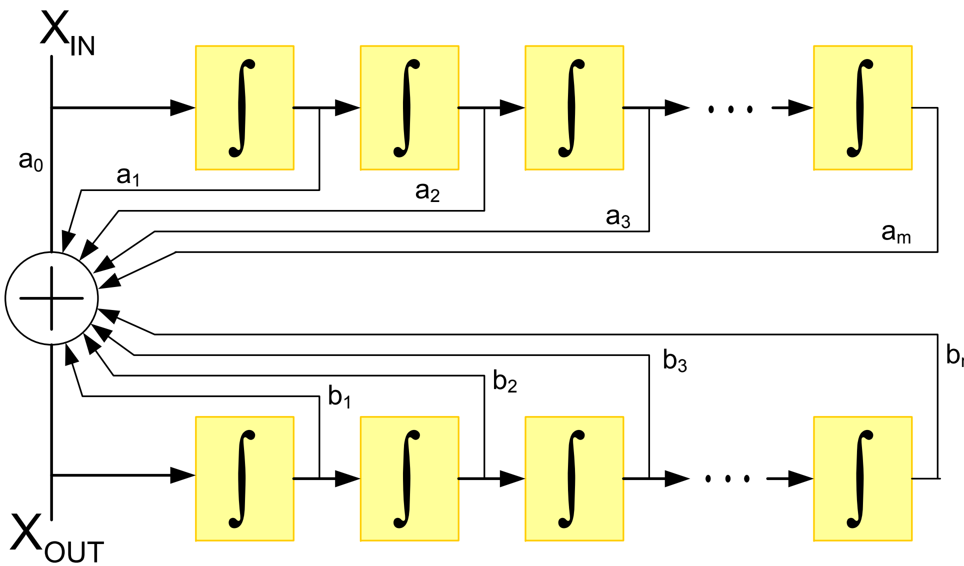
Review from Last Time

Applications of integrators to solving differential equations



Consider the standard integral form

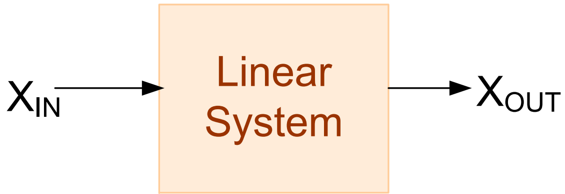
$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$



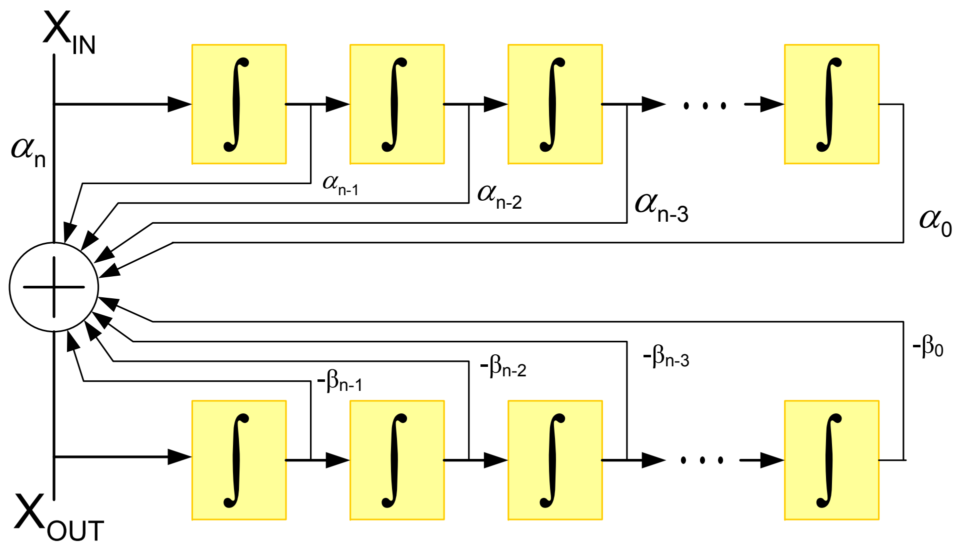
This circuit is comprised of summers and integrators
Can solve an arbitrary linear differential equation
This concept was used in Analog Computers in the past

Review from Last Time

Applications of integrators to filter design



$$T(s)=\frac{\alpha_n s^n+\alpha_{m-1}s^{n-1}+\ldots\alpha_1 s\ +\alpha_0}{s^n+\beta_{n-1}s^{n-1}+\ldots+\beta_1 s+\beta_0}$$

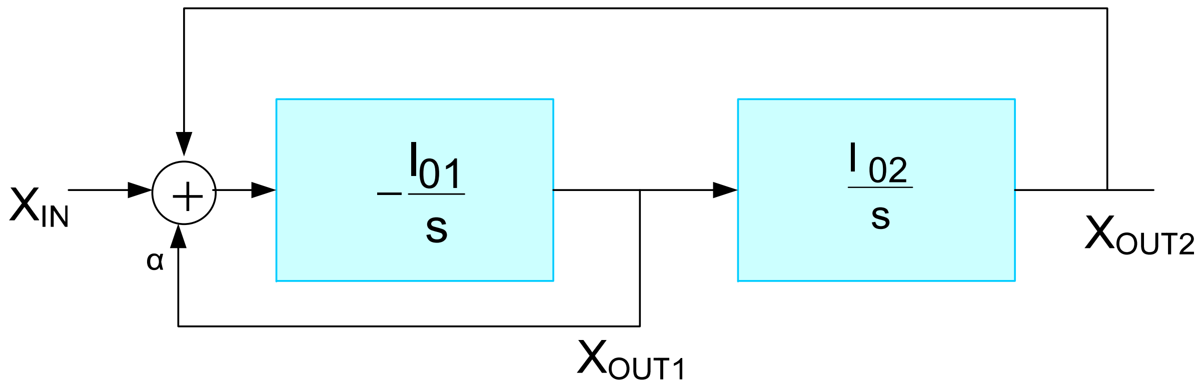


Can design (synthesize) any T(s) with just integrators and summers !

Integrators are not used "open loop" so loss is not added

Although this approach to filter design works, often more practical methods are used

Applications of integrators to filter design



This is a two-integrator-loop filter

$$X_{OUT1}=\left(-\frac{I_{01}}{s}\right)\left(X_{IN}+X_{OUT2}+\alpha X_{OUT1}\right)$$

$$X_{OUT2}=\left(\frac{I_{02}}{s}\right)X_{OUT1}$$

$$\frac{X_{OUT1}}{X_{IN}}=T_1(s)=\frac{-I_{01}s}{s^2+\alpha I_{01}s+I_{01}I_{02}}$$

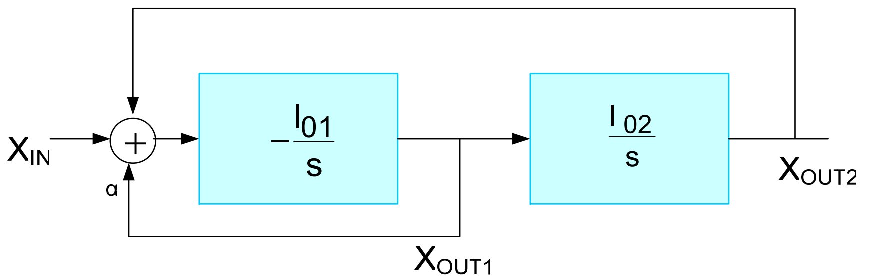
$$\frac{X_{OUT2}}{X_{IN}}=T_2(s)=\frac{-I_{01}I_{02}}{s^2+\alpha I_{01}s+I_{01}I_{02}}$$

These are 2-nd order filters

If $I_{01}=I_{02}=I_0$, these transfer functions reduce to

$$T_1(s)=\frac{-I_0 s}{s^2+\alpha I_0 s+I_0^2} \qquad T_2(s)=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Applications of integrators to filter design



$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

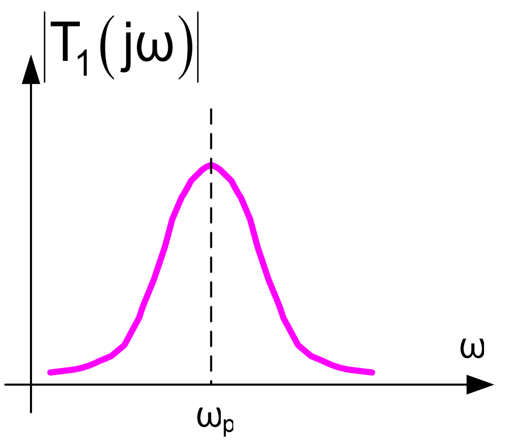
$$T_2(s) = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2}$$

$$I_{01} = I_{02} = I_0$$

Consider $T_1(j\omega)$

$$T_1(j\omega) = \frac{-j\omega I_0}{\left(I_0^2 - \omega^2\right) + j\omega\alpha I_0}$$

$$\left|T_1(j\omega)\right| = \frac{\omega I_0}{\sqrt{\left(I_0^2 - \omega^2\right)^2 + \left(\omega\alpha I_0\right)^2}}$$



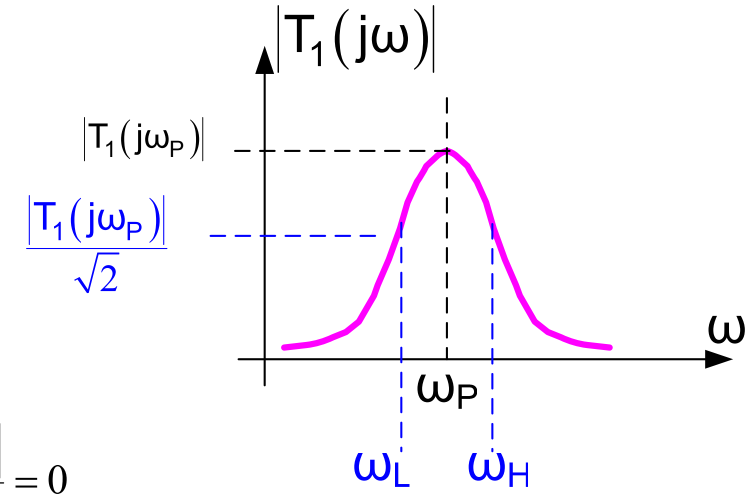
This is the standard 2nd order bandpass transfer function

Now lets determine the BW and ω_P

Applications of integrators to filter design

Determine the BW and ω_P

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$



$$\left|T_1(j\omega)\right| = \frac{\omega I_0}{\sqrt{\left(I_0^2-\omega^2\right)^2 + \left(\omega \alpha I_0\right)^2}}$$

To determine ω_P, must set $\dfrac{d\left|T_1(j\omega)\right|}{d\omega} = 0$

This will occur also when $\dfrac{d\left|T_1(j\omega)\right|^2}{d\omega^2} = 0$ and the latter is easier to work with

$$\left|T_1(j\omega)\right|^2 = \frac{\omega^2 I_0^2}{\left(I_0^2-\omega^2\right)^2 + \left(\omega \alpha I_0\right)^2}$$

$$\frac{d\left|T_1(j\omega)\right|^2}{d\omega^2} = \frac{\left(\left(I_0^2-\omega^2\right)^2 + \left(\omega \alpha I_0\right)^2\right) I_0^2 - \omega^2 I_0^2 \left(-2\left(I_0^2-\omega^2\right) + \left(\alpha I_0\right)^2\right)}{\left[\left(I_0^2-\omega^2\right)^2 + \left(\omega \alpha I_0\right)^2\right]^2} = 0$$

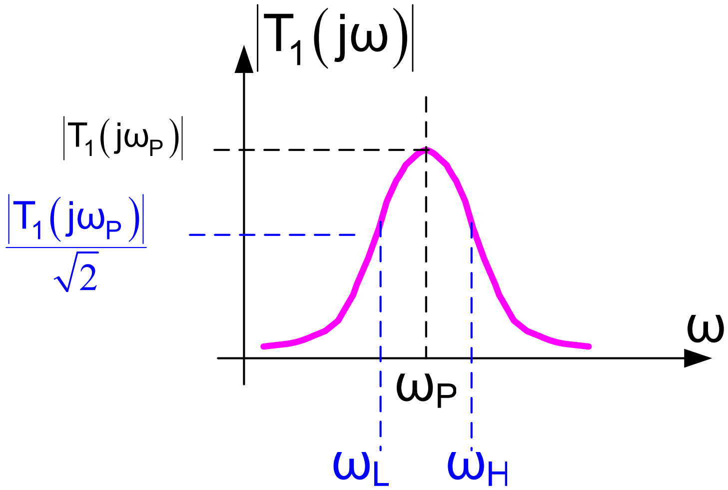
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s)=\frac{-I_0 s}{s^2+\alpha I_0 s+I_0^2} \qquad \left|T_1(j\omega)\right|=\frac{\omega I_0}{\sqrt{\left(I_0^2-\omega^2\right)^2+\left(\omega\alpha I_0\right)^2}}$$

$$\frac{d\left|T_1(j\omega)\right|^2}{d\omega^2}=\frac{\left(\left(I_0^2-\omega^2\right)^2+\left(\omega\alpha I_0\right)^2\right)I_0^2-\omega^2 I_0^2\left(-2\left(I_0^2-\omega^2\right)+\left(\alpha I_0\right)^2\right)}{\left[\left(I_0^2-\omega^2\right)^2+\left(\omega\alpha I_0\right)^2\right]^2}=0$$



It suffices to set the numerator to 0

$$\left(\left(I_0^2-\omega^2\right)^2+\left(\omega\alpha I_0\right)^2\right)I_0^2=\omega^2 I_0^2\left(-2\left(I_0^2-\omega^2\right)+\left(\alpha I_0\right)^2\right)$$

Solving, we obtain

$$\omega_P = I_0$$

Substituting back into the magnitude expression, we obtain

$$\left|T_1(j\omega_P)\right|=\frac{I_0 I_0}{\sqrt{\left(I_0^2-I_0^2\right)+\left(I_0\alpha\right)^2}}=\frac{1}{\alpha}$$

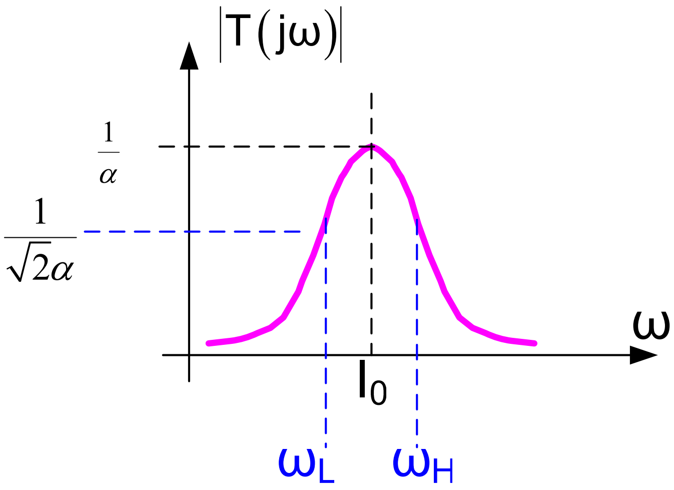
Although the analysis is somewhat tedious, the results are clean

Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2} \qquad \left|T_1(j\omega)\right| = \frac{\omega I_0}{\sqrt{\left(I_0^2 - \omega^2\right)^2 + \left(\omega \alpha I_0\right)^2}}$$



To obtain ω_L and ω_H, must solve $\quad \left|T_1(j\omega)\right| = \frac{1}{\sqrt{2}\alpha}$

This becomes

$$\frac{1}{2\alpha^2} = \frac{\left(\left(I_0^2 - \omega^2\right)^2 + \left(\omega \alpha I_0\right)^2\right) I_0^2 - \omega^2 I_0^2 \left(-2\left(I_0^2 - \omega^2\right) + \left(\alpha I_0\right)^2\right)}{\left[\left(I_0^2 - \omega^2\right)^2 + \left(\omega \alpha I_0\right)^2\right]^2}$$

The expressions for ω_L and ω_H can be easily obtained but are somewhat messy, but from these expressions, we obtain the simple expressions

$$BW = \omega_H - \omega_L = \alpha I_0$$

$$\sqrt{\omega_H \omega_L} = I_0$$

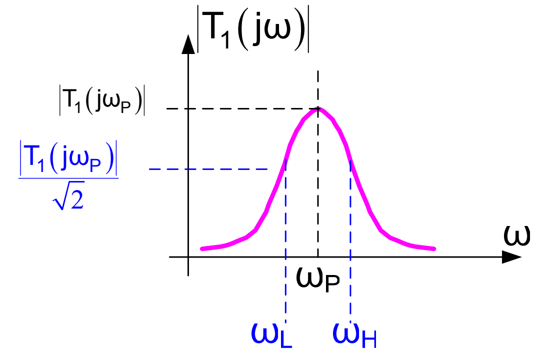
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

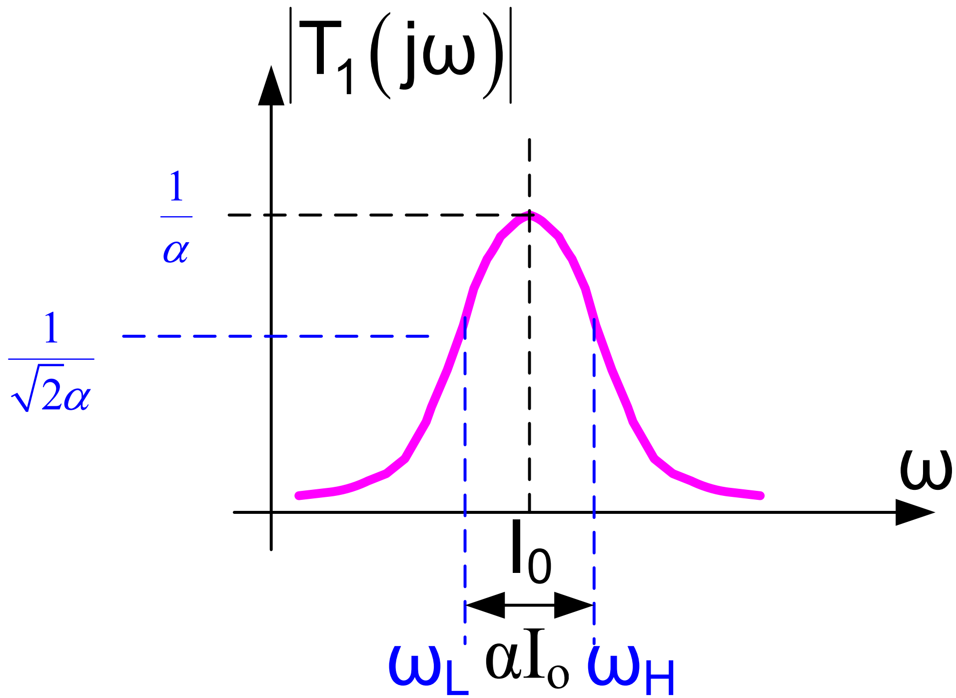
$$\left|T_1(j\omega)\right| = \frac{\omega I_0}{\sqrt{\left(I_0^2 - \omega^2\right)^2 + \left(\omega \alpha I_0\right)^2}}$$



$$\omega_P = I_0$$

$$BW = \alpha I_0$$

$$\left|T_1(j\omega_P)\right| = \frac{1}{\alpha}$$



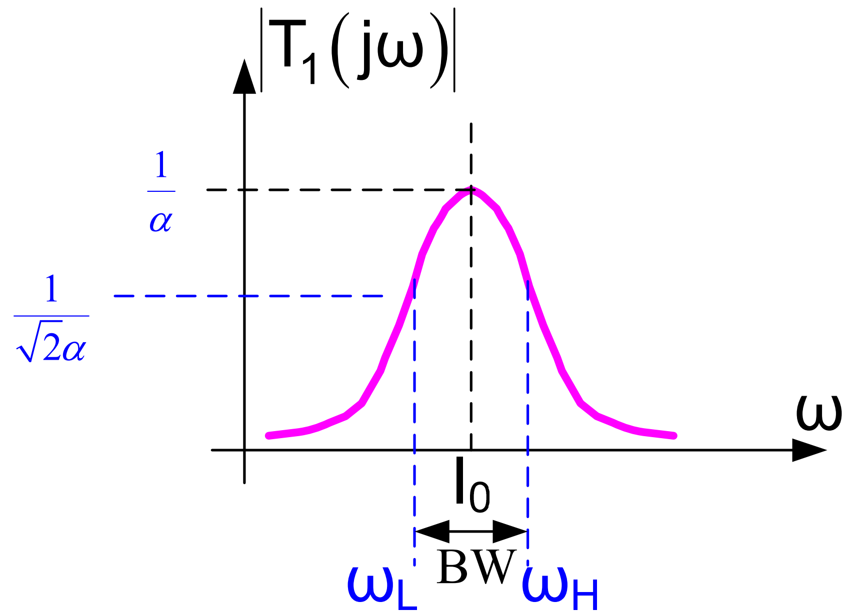
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

$$BW = \alpha I_0 \qquad \sqrt{\omega_H \omega_L} = I_0$$



Often express the standard 2nd order bandpass transfer function as

$$T_1(s) = \frac{-I_0 s}{s^2 + BWs + I_0^2}$$

Applications of integrators to filter design

The 2nd order Bandpass Filter

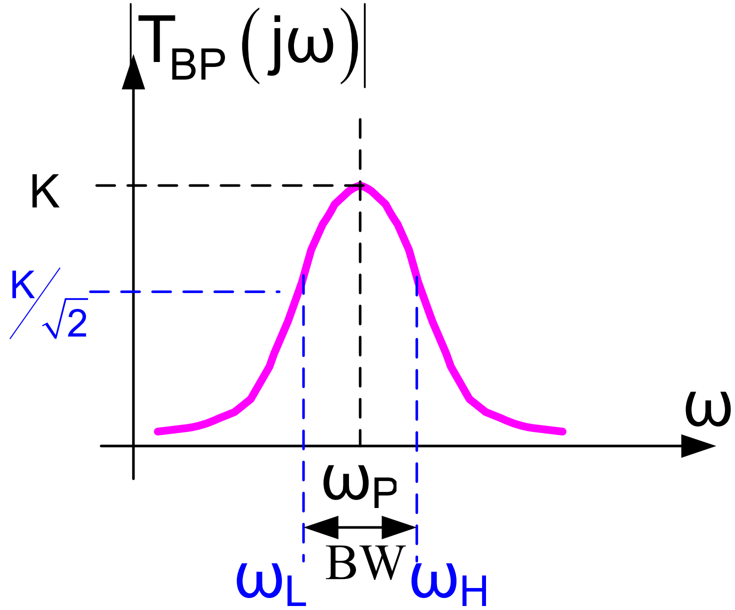
These results can be generalized

$$T_{BP}(s) = \frac{Hs}{s^2 + as + b}$$

$$BW = a$$

$$\omega_P = \sqrt{b}$$

$$K = \frac{|H|}{a}$$



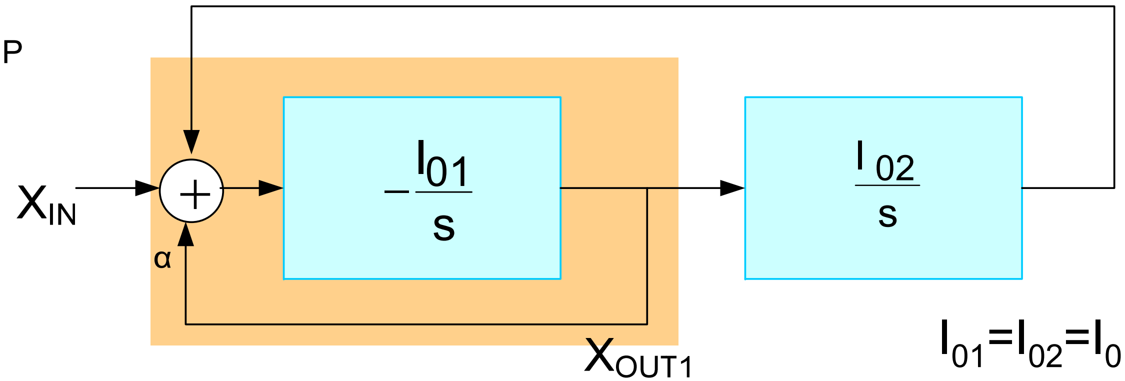
Applications of integrators to filter design

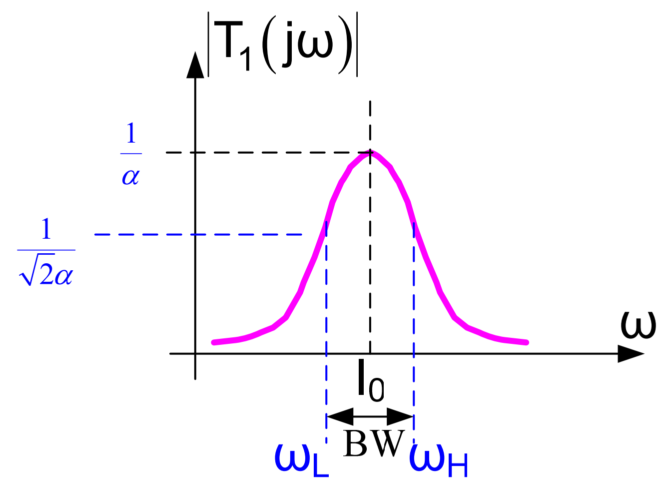
The 2nd order Bandpass Filter

Determine the BW and ω_P

$$T_1(s)=\frac{-I_0 s}{s^2+\alpha I_0 s+I_0^2}$$

$$T_1(s)=\frac{-I_0 s}{s^2+BWs+I_0^2}$$





Can readily be implemented with a summing inverting integrator and a noninverting integrator

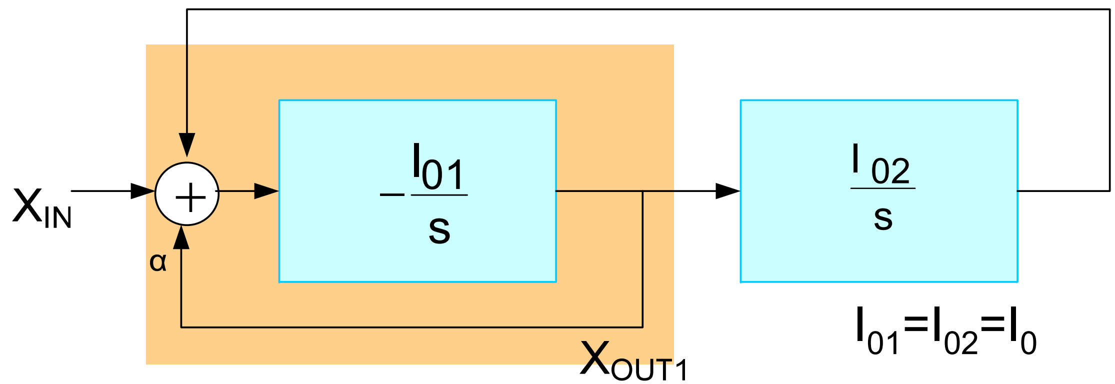
Applications of integrators to filter design

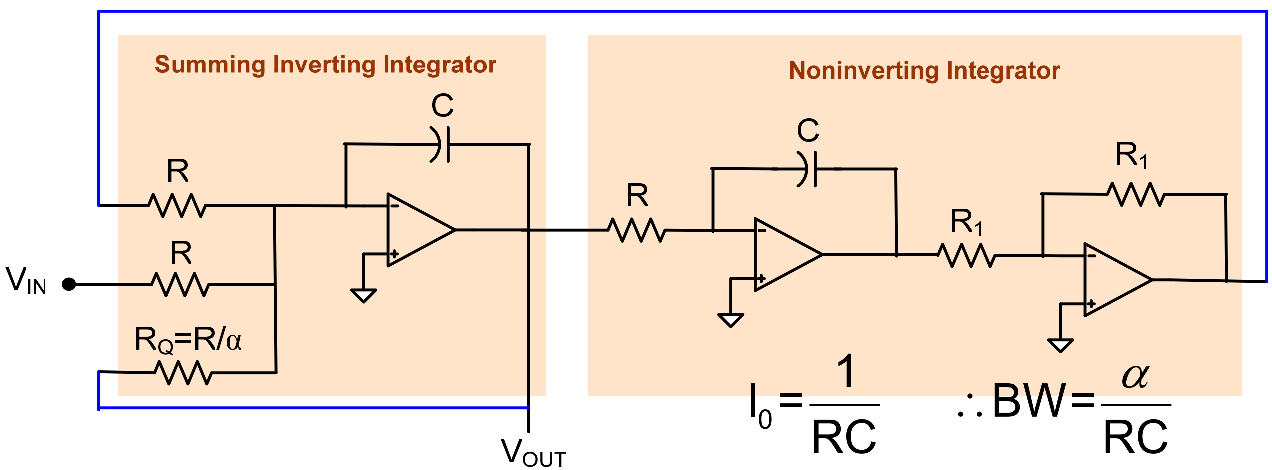
The 2nd order Bandpass Filter

Determine the BW and ω_P

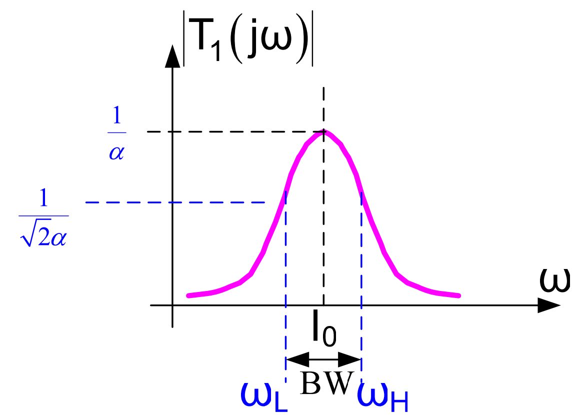
$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

$$T_1(s) = \frac{-I_0 s}{s^2 + BW s + I_0^2}$$





$$I_0 = \frac{1}{RC} \qquad \therefore BW = \frac{\alpha}{RC}$$



$$\omega_P = I_0$$

$$BW = \alpha I_0$$

- Widely used 2nd order Bandpass Filter
- BW can be adjusted with R_Q
- Peak gain changes with R_Q
- Note no loss is added to the integrators

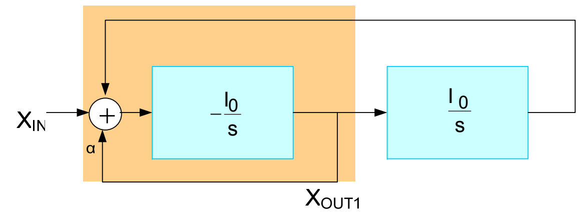
Applications of integrators to filter design

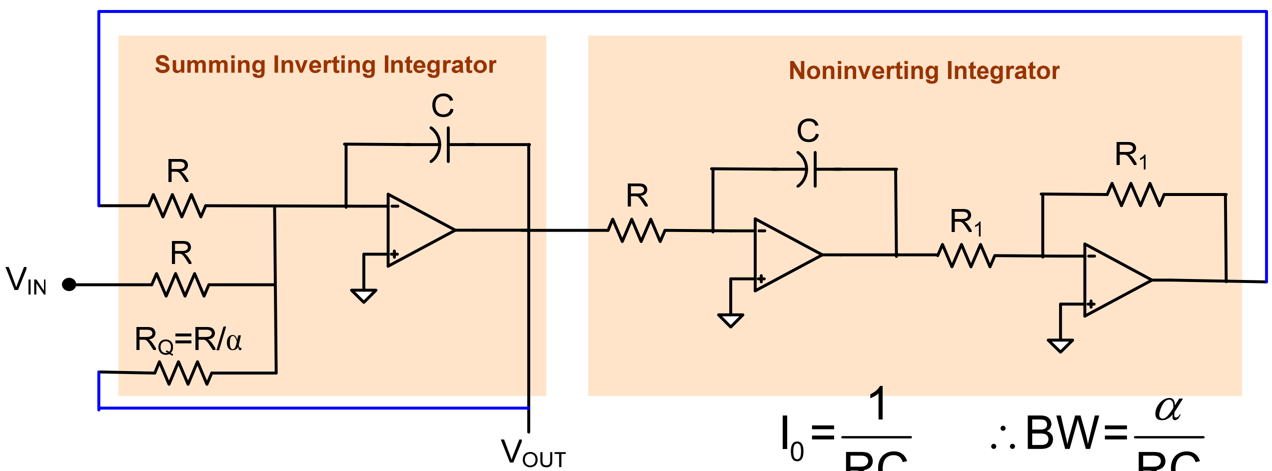
The 2nd order Bandpass Filter

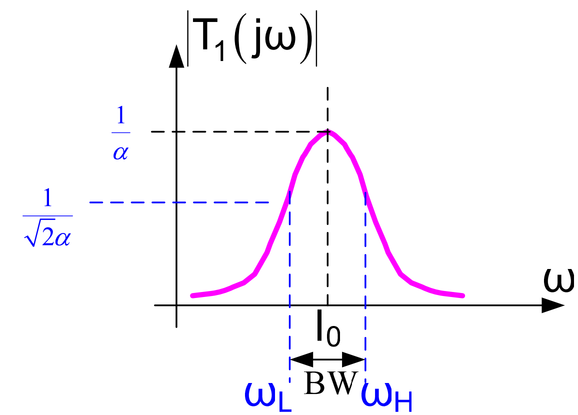
Design Strategy

Assume BW and ω_P are specified

$$T_{BP}(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$







$$I_0 = \frac{1}{RC} \qquad \therefore BW = \frac{\alpha}{RC}$$

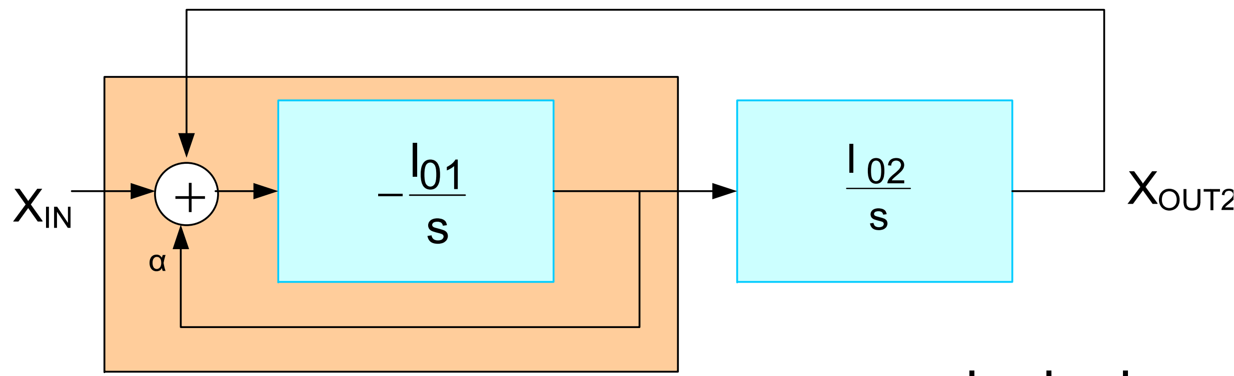
$$\omega_P = I_0$$

1. Pick C (use some practical or convenient value)
2. Solve expression $\omega_P = \frac{1}{RC}$ to obtain R
3. Solve expression $BW = \frac{\alpha}{RC}$ to obtain α and thus R_Q

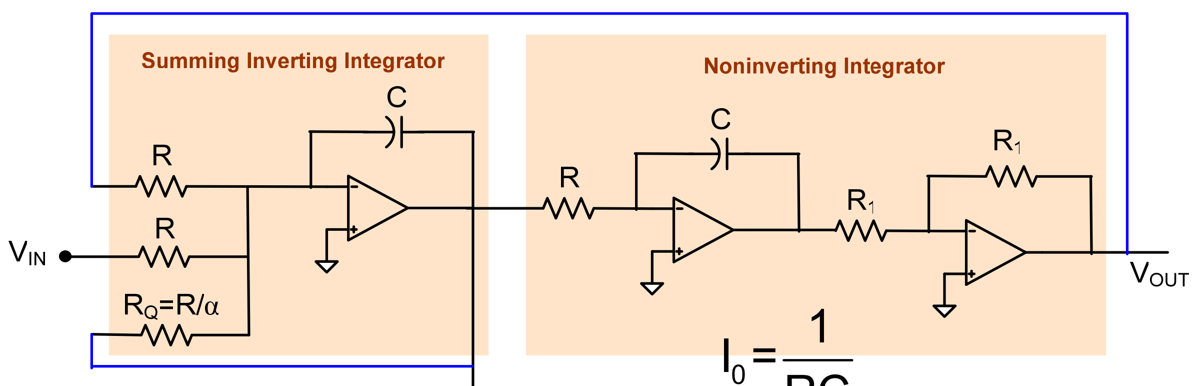
Applications of integrators to filter design

The 2nd order Lowpass Filter

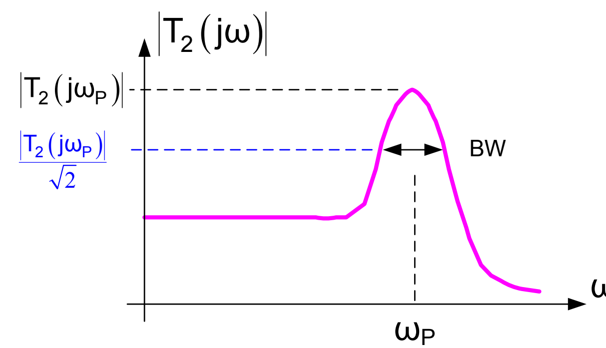
$$T_2(s)=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$



$$I_{01}=I_{02}=I_0$$



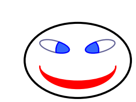
$$I_0=\frac{1}{RC}$$



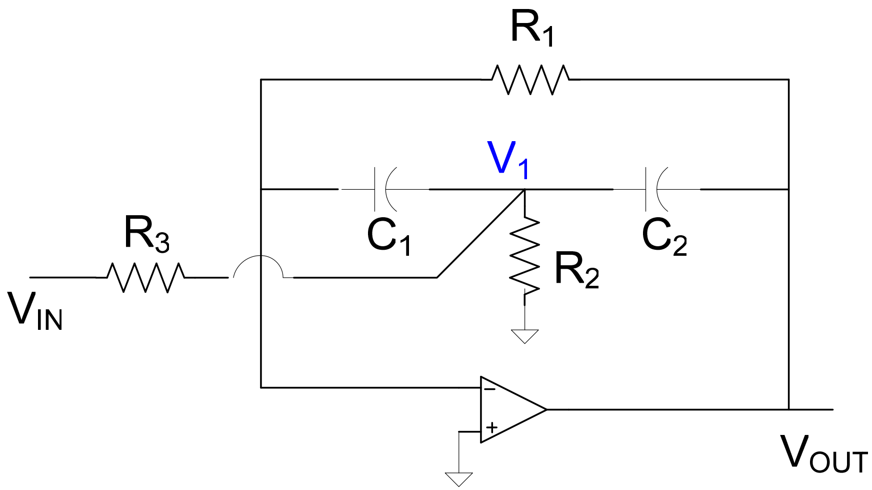
Exact expressions for BW and ω_P are very complicated but $\omega_P \approx I_0$

- Widely used 2nd order Lowpass Filter
- BW can be adjusted with R_Q but expression not so simple
- Peak gain changes with R_Q
- Note no loss is added to the integrators

Design procedure to realize a given 2nd order lowpass function is straightforward



Another 2nd-order Bandpass Filter



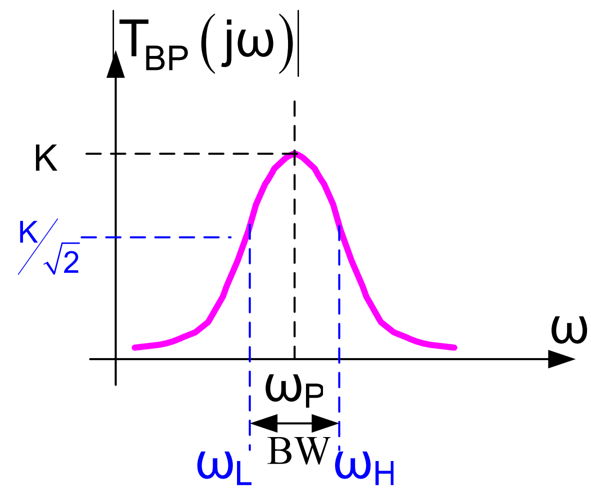
$$V_1\left(sC_1+sC_2+G_2+G_3\right) = V_{OUT}sC_2+V_{IN}G_3$$

$$V_1sC_1+V_{OUT}G_1 = 0$$

$$T(s) = -\frac{\frac{s}{R_3C_2}}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_1C_2}\right)+\frac{1}{(R_2//R_3)R_1C_1C_2}}$$

If the capacitors are matched and equal to C

$$T(s) = -\frac{\frac{s}{R_3C}}{s^2+s\left(\frac{2}{R_1C}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$



Since this is of the general form of a 2nd order BP transfer function, obtain

$$\omega_P=\frac{1}{\sqrt{R_1(R_2//R_3)}C}$$

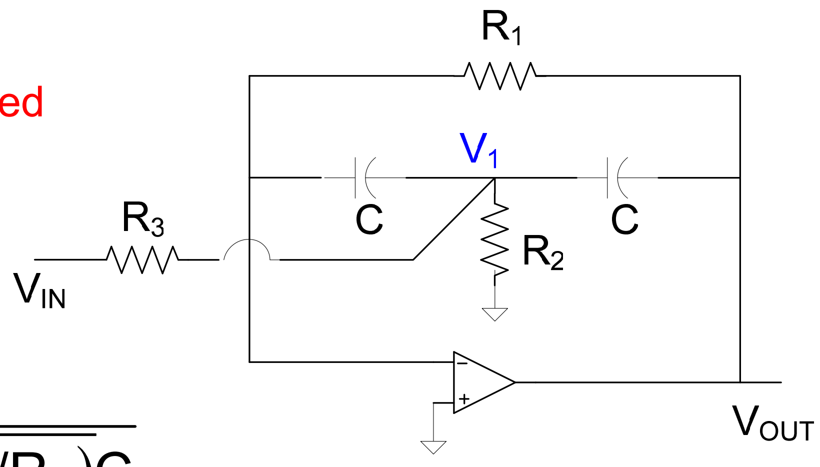
$$BW = \frac{2}{R_1C} \qquad K=\frac{R_1}{2R_3}$$

Another 2nd-order Bandpass Filter

Design Strategy

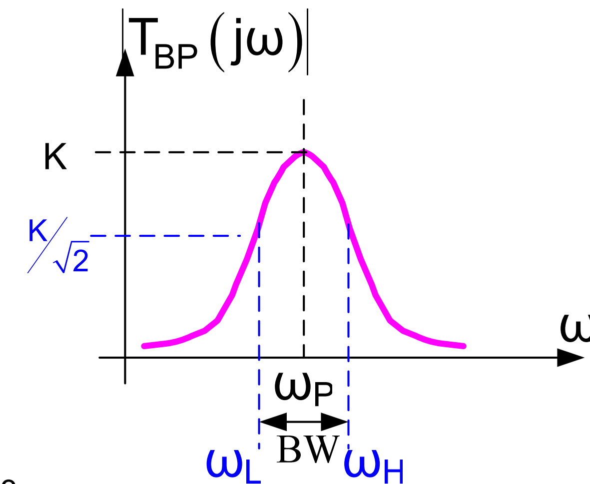
Assume BW, ω_P , and K are specified

$$T(s) = -\frac{\frac{s}{R_3C}}{s^2+s\left(\frac{2}{R_1C}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$



$$BW = \frac{2}{R_1C} \qquad \omega_P = \frac{1}{\sqrt{R_1(R_2//R_3)C}}$$

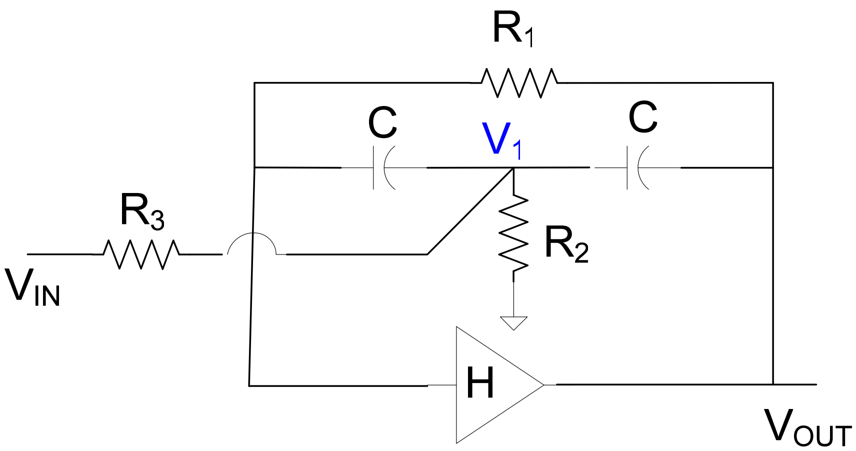
$$K = \frac{R_1}{2R_3}$$



1. Pick C to some practical or convenient value
2. Solve expression $BW = \frac{2}{R_1C}$ to obtain R_1
3. Solve expression $K = \frac{R_1}{2R_3}$ to obtain α and thus R_3
4. Solve expression $\omega_P = \frac{1}{\sqrt{R_1(R_2//R_3)C}}$ to obtain R_2

Another 2nd-order Bandpass Filter

Termed the “STAR” biquad by inventors at Bell Labs



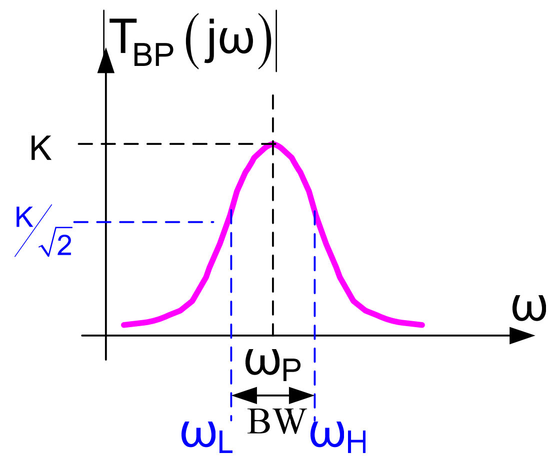
$$\left.\begin{aligned} &V_1\left(sC+sC+G_2+G_3\right) = V_{OUT}sC+V_{IN}G_3+V_{OUT}\frac{sC}{H} \\ &\frac{V_{OUT}}{H}\left(sC+G_1\right)=V_1sC+V_{OUT}G_1 \end{aligned}\right\}$$

$$T(s) = -\frac{\frac{s}{R_3C}\left(\frac{H}{H-1}\right)}{s^2+s\left(\frac{2}{R_1C}-\frac{1}{(R_2//R_3)(H-1)}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$

$$\omega_P=\frac{1}{\sqrt{R_1(R_2//R_3)C}}$$

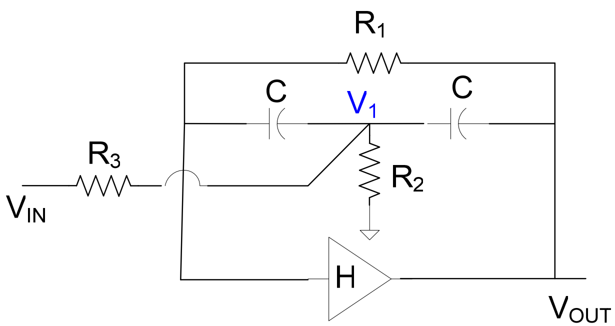
$$BW = \frac{2}{R_1C}-\frac{1}{(R_2//R_3)(H-1)}$$

$$K = \frac{\frac{1}{R_3}\left(\frac{H}{H-1}\right)}{\left(\frac{2}{R_1}-\frac{1}{(R_2//R_3)(H-1)}\right)}$$



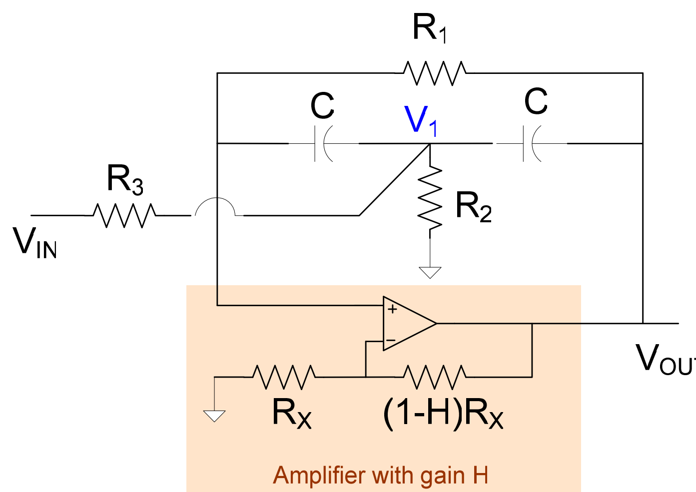
For the appropriate selection of component values, this is one of the best 2nd order bandpass filters that has been published

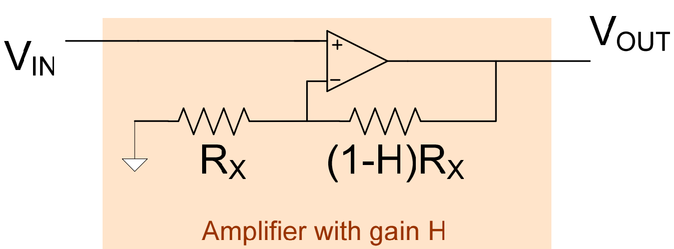
STAR 2nd-order Bandpass Filter



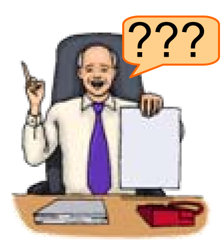
$$T(s) = -\frac{\frac{s}{R_3C}\left(\frac{H}{H-1}\right)}{s^2+s\left(\frac{2}{R_1C}-\frac{1}{(R_2//R_3)(H-1)}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$

Implementation:

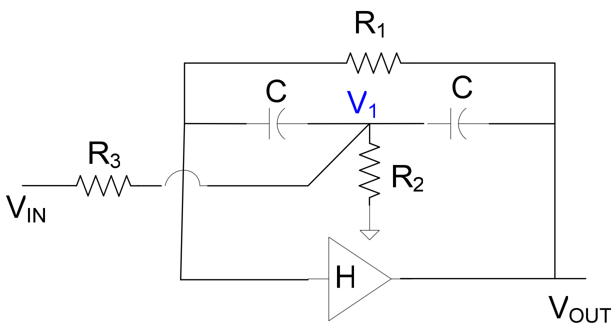




But the filter doesn't work !

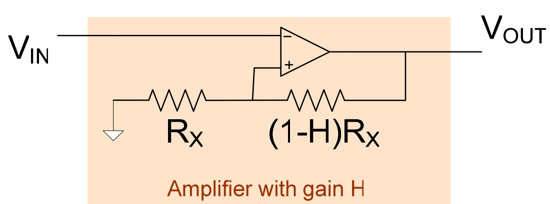


STAR 2nd-order Bandpass Filter

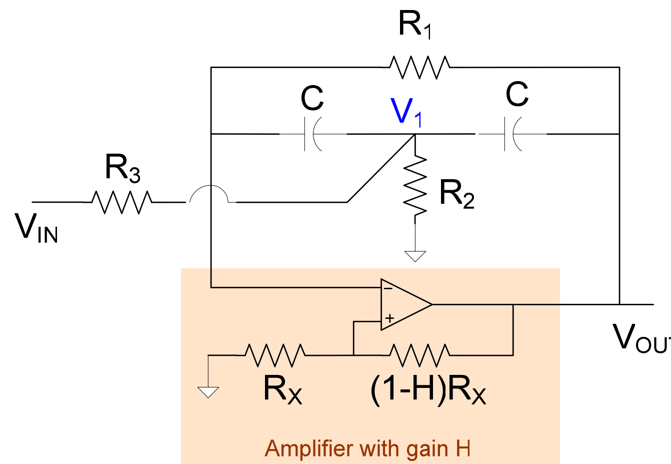


$$T(s) = -\frac{\frac{s}{R_3C}\left(\frac{H}{H-1}\right)}{s^2+s\left(\frac{2}{R_1C}-\frac{1}{(R_2//R_3)(H-1)}\right)+\frac{1}{(R_2//R_3)R_1C^2}}$$

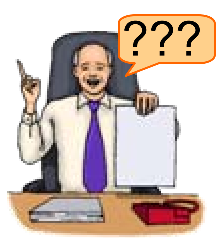
Implementation:



If op amp ideal, $\frac{V_{OUT}}{V_{IN}} = H$



Works fine !



Will discuss why this happens later!

Reduces to previous bandpass filter at H gets large

Note that the “H” amplifier has feedback to positive terminal

End of Lecture 14